CALCULATION OF RADIATIVE HEAT TRANSFER WITH ANISOTROPIC DISPERSION OF RADIANT ENERGY

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Opticogeometrical resolvents and their equations are established for the case of anisotropy in the optical properties of a medium and its surfaces.

It is well known that the optical properties of all materials depend very strongly on the direction of incident and transmitted rays, the latter having an appreciable effect on the resultant radiant fluxes [1]. The advantages of thermal radiation cannot always be fully utilized in modern technological processes, meanwhile, because no complete general theory has been developed yet on calculating the radiative heat transfer where the optical properties of solid surfaces and semitranslucent media are anisotropic [1-8]. This study here is concerned with an analysis of the problem.

It is assumed here that there occurs no spectral redistribution of radiant energy due to dispersion. In the case of a closed system which consists of anisotropically radiating and dispersing boundary surfaces and media contained between them, we obtain the following integral equations for the luminance of effective radiation from elements of boundary surfaces and of the media contained between them:

$$B_e\left(\frac{M}{r_0}\right)=B_c\left(\frac{M}{r_0}\right)+\int_{F_1}B_e\left(\frac{c_1}{r(c_1M)}\right)R\left(\frac{M}{r(c_1M)}\right)\gamma\times\left(\frac{M}{r(c_1M)\,r_0}\right)K_1(c_1M)\exp\left(\int_{r(c_1)}^{r(M)}-k(r)\,dr\right)dF(c_1)$$
$$+\int_{V_1}B_e\left(\frac{n_1}{r(n_1M)}\right)R\left(\frac{M}{r(n_1M)}\right)\gamma\left(\frac{M}{r(n_1M)\,r_0}\right)K_2(n_1M)\exp\left(\int_{r(n_1)}^{r(M)}-k(r)\,dr\right)dV(n_1), \quad (1)$$

$$B_e\left(\frac{b}{r_0}\right)=B_c\left(\frac{b}{r_0}\right)+\int_{F_1}B_e\left(\frac{c_1}{r(c_1b)}\right)\beta\left(\frac{b}{r(c_1b)}\right)\gamma\times\left(\frac{b}{r(c_1b)\,r_0}\right)K_3(c_1b)\exp\left(\int_{r(c_1)}^{r(b)}-k(r)\,dr\right)dF(c_1)$$
$$+\int_{V_1}B_e\left(\frac{b}{r(n_1b)}\right)\beta\left(\frac{b}{r(n_1b)}\right)\gamma\left(\frac{b}{r(n_1b)\,r_0}\right)K_4(n_1b)\exp\left(\int_{r(n_1)}^{r(b)}-k(r)\,dr\right)dV(n_1), \quad (2)$$

where

$$K_1(c_1M)=\cos\left(\widehat{\bar n(c_1)\bar r(c_1M)}\right)\cos\left(\widehat{\bar r(c_1M)\bar n(M)}\right)[\pi r^2(c_1M)]^{-1}, \quad (3)$$

$$K_2(n_1M)=\cos\left(\widehat{\bar r(n_1M)\bar n(M)}\right)[\pi r^2(c_1M)]^{-1}, \quad (4)$$

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$$K_3(c_1b) = \cos\left(\widehat{\bar{n}(c_1)\,\bar{r}(c_1b)}\right)[4\pi r^2(c_1b)]^{-1}, \tag{5}$$

$$K_4(n_1b) = [4\pi r^2(n_1b)]^{-1}. \tag{6}$$

The solution of Eqs. (1), (2) yields the following expression for the luminance of effective radiation along any direction r_0 from elements of boundary surfaces and the media contained between them:

$$\begin{aligned} B_e\left(\frac{M}{r_0}\right) &= B_c\left(\frac{M}{r_0}\right) + \int\limits_{F_1} B_c\left(\frac{c_1}{r(c_1M)}\right) R\left(\frac{M}{r(c_1M)}\right)\gamma \\ &\times\left(\frac{M}{r(c_1M)\,r_0}\right) K_1(c_1M)\exp\left(\int\limits_{r(c_1)}^{r(M)} -k(r)\,dr\right) dF(c_1) \\ &+ \iint\limits_{F_1F_2} B_c\left(\frac{c_1}{r(c_1c_2)}\right)\mathcal{P}_{11}\left(\frac{c_1c_2M}{r(c_1c_2)\,r(c_2M)\,r_0}\right) dF(c_1)\,dF(c_2) \\ &+ \iint\limits_{F_1V_2} B_c\left(\frac{c_1}{r(c_1n_2)}\right)\mathcal{P}_{32}\left(\frac{c_1n_2M}{r(c_1n_2)\,r(n_2M)\,r_0}\right) dF(c_1)\,dV(n_2) \\ &+ \int\limits_{V_1} B_c\left(\frac{n_1}{r(n_1M)}\right) R\left(\frac{M}{r(c_1M)}\right)\gamma\left(\frac{M}{r(c_1M)\,r_0}\right) K_2(n_1M)\exp\left(\int\limits_{r(n_1)}^{r(M)} -k(r)\,dr\right) dV(n_1 \\ &+ \iint\limits_{V_1F_2} B_c\left(\frac{n_1}{r(n_1c_2)}\right)\mathcal{P}_{21}\left(\frac{n_1c_2M}{r(n_1c_2)\,r(c_2M)\,r_0}\right) dV(n_1)\,dF(c_2) + \\ &+ \iint\limits_{V_1V_2} B_c\left(\frac{n_1}{r(n_1n_2)}\right)\mathcal{P}_{42}\left(\frac{n_1n_2M}{r(n_1n_2)\,r(n_2M)\,r_0}\right) dV(n_1)\,dV(n_2), \end{aligned} \tag{7}$$

$$\begin{aligned} B_e\left(\frac{b}{r_0}\right) &= B_c\left(\frac{b}{r_0}\right) + \int\limits_{F_1} B_c\left(\frac{c_1}{r(c_1b)}\right)\beta\left(\frac{b}{r(c_1b)}\right)\gamma \\ &\times\left(\frac{b}{r(c_1b)\,r_0}\right) K_3(c_1b)\exp\left(\int\limits_{r(c_1)}^{r(b)} -k(r)\,dr\right) dF(c_1) \\ &+ \iint\limits_{F_1F_2} B_c\left(\frac{c_1}{r(c_1c_2)}\right)\mathcal{P}_{13}\left(\frac{c_1c_2b}{r(c_1c_2)\,r(c_2b)\,r_0}\right) dF(c_1)\,dF(c_2) \\ &+ \iint\limits_{F_1V_2} B_c\left(\frac{c_1}{r(c_1n_2)}\right)\mathcal{P}_{34}\left(\frac{c_1n_2b}{r(c_1n_2)\,r(n_2b)\,r_0}\right) dF(c_1)\,dV(n_2) \\ &+ \int\limits_{V_1} B_c\left(\frac{n_1}{r(n_1b)}\right)\beta\left(\frac{b}{r(n_1b)}\right)\gamma\left(\frac{b}{r(n_1b)\,r_0}\right) K_4(n_1b)\exp\left(\int\limits_{r(n_1)}^{r(b)} -k(r)\,dr\right) dV(n_1) \\ &+ \iint\limits_{V_1F_2} B_c\left(\frac{n_1}{r(n_1c_2)}\right)\mathcal{P}_{23}\left(\frac{n_1c_2b}{r(n_1c_2)\,r(c_2b)\,r_0}\right) dV(n_1)\,dF(c_2) \\ &+ \iint\limits_{V_1V_2} B_c\left(\frac{n_1}{r(n_1n_2)}\right)\mathcal{P}_{44}\left(\frac{n_1n_2b}{r(n_1n_2)\,r(n_2b)\,r_0}\right) dV(n_1)\,dV(n_2). \end{aligned} \tag{8}$$

The opticogeometrical functions $\mathcal{P}_{ik}$ represent the effect of innumerable dispersions of radiant energy at the boundary surfaces and inside the volume of the media in a system, and they are determined from the following system of opticogeometrical integral equations:

a. for boundary points on the surfaces

$$\begin{aligned} \mathcal{P}_{11}\left(\frac{c_1c_2M}{r(c_1c_2)\,r(c_2M)\,r_0}\right) &= K'_{11}\left(\frac{c_1c_2M}{r(c_1c_2)\,r(c_2M)\,r_0}\right) \\ &+ \int\limits_{F_3} K'_1(c_1c_2)\,\mathcal{P}_{11}\left(\frac{c_2c_3M}{r(c_2c_3)\,r(c_3M)\,r_0}\right) dF(c_3) \end{aligned} \tag{9}$$

$$+\int\limits_{V_3} K_1'(c_1c_2)\,\mathscr{P}_{32}\left(\frac{c_2n_3M}{(r(c_2n_3)\,r(n_3M)\,r_0}\right)dV(n_3), \tag{9}$$

$$\mathscr{P}_{32}\left(\frac{c_1n_2M}{r(c_1n_2)\,r(n_2M)\,r_0}\right)=K_{32}'\left(\frac{c_1n_2M}{r(c_1n_2)\,r(n_2M)\,r_0}\right)$$

$$+\int\limits_{F_3} K_3'(c_1n_2)\,\mathscr{P}_{21}\left(\frac{n_2c_3M}{r(n_2c_3)\,r(c_3M)\,r_0}\right)dF(c_3)$$

$$+\int\limits_{V_3} K_3'(c_1n_2)\,\mathscr{P}_{42}\left(\frac{n_2n_3M}{r(n_2n_3)\,r(n_3M)\,r_0}\right)dV(n_3), \tag{10}$$

$$\mathscr{P}_{21}\left(\frac{n_1c_2M}{r(n_1c_2)\,r(c_2M)\,r_0}\right)=K_{21}'\left(\frac{n_1c_2M}{r(n_1c_2)\,r(c_2M)\,r_0}\right)$$

$$+\int\limits_{F_3} K_2'(n_1c_2)\,\mathscr{P}_{11}\left(\frac{c_2c_3M}{r(c_2c_3)\,r(c_3M)\,r_0}\right)dF(c_3)$$

$$+\int\limits_{V_3} K_2'(n_1c_2)\,\mathscr{P}_{32}\left(\frac{c_2n_3M}{r(c_2n_3)\,r(n_3M)\,r_0}\right)dV(n_3), \tag{11}$$

$$\mathscr{P}_{42}\left(\frac{n_1n_2M}{r(n_1n_2)\,r(n_2M)\,r_0}\right)=K_{42}'\left(\frac{n_1n_2M}{r(n_1n_2)\,r(n_2M)\,r_0}\right)$$

$$+\int\limits_{F_3} K_4'(n_1n_2)\,\mathscr{P}_{21}\left(\frac{n_2c_3M}{r(n_2c_3)\,r(c_3M)\,r_0}\right)dF(c_3)$$

$$+\int\limits_{V_3} K_4'(n_1n_2)\,\mathscr{P}_{42}\left(\frac{n_2n_3M}{r(n_2n_3)\,r(n_3M)\,r_0}\right)dV(n_3); \tag{12}$$

b. for points inside the volume of a medium

$$\mathscr{P}_{13}\left(\frac{c_1c_2b}{r(c_1c_2)\,r(c_2b)\,r_0}\right)=K_{13}'\left(\frac{c_1c_2b}{r(c_1c_2)\,r(c_2b)\,r_0}\right)$$

$$+\int\limits_{F_3} K_1'(c_1c_2)\,\mathscr{P}_{13}\left(\frac{c_2c_3b}{r(c_2c_3)\,r(c_3b)\,r_0}\right)dF(c_3)$$

$$+\int\limits_{V_3} K_1(c_1c_2)\,\mathscr{P}_{34}\left(\frac{c_2n_3b}{r(c_2n_3)\,r(n_3b)\,r_0}\right)dV(n_3), \tag{13}$$

$$\mathscr{P}_{34}\left(\frac{c_1n_2b}{r(c_1n_2)\,r(n_2b)\,r_0}\right)=K_{34}'\left(\frac{c_1n_2b}{r(c_1n_2)\,r(n_2b)\,r_0}\right)$$

$$+\int\limits_{F_3} K_3'(c_1n_2)\,\mathscr{P}_{23}\left(\frac{n_2c_3b}{r(n_2c_3)\,r(c_3b)\,r_0}\right)dF(c_3)$$

$$+\int\limits_{V_3} K_3'(c_1n_2)\,\mathscr{P}_{44}\left(\frac{n_2n_3b}{r(n_2n_3)\,r(n_3b)\,r_0}\right)dV(n_3), \tag{14}$$

$$\mathscr{P}_{23}\left(\frac{n_1c_2b}{r(n_1c_2)\,r(c_2b)\,r_0}\right)=K_{23}'\left(\frac{n_1c_2b}{r(n_1c_2)\,r(c_2b)\,r_0}\right)$$

$$+\int\limits_{F_3} K_2'(n_1c_2)\,\mathscr{P}_{13}\left(\frac{c_2c_3b}{r(c_2c_3)\,r(c_3b)\,r_0}\right)dF(c_3)$$

$$+\int\limits_{V_3} K_2'(n_1c_2)\,\mathscr{P}_{34}\left(\frac{c_2n_3b}{r(c_2n_3)\,r(n_3b)\,r_0}\right)dV(n_3), \tag{15}$$

$$\mathscr{P}_{44}\left(\frac{n_1n_2b}{r(n_1n_2)\,r(n_2b)\,r_0}\right)=K_{44}'\left(\frac{n_1n_2b}{r(n_1n_2)\,r(n_2b)\,r_0}\right)$$

$$+\int_{F_2} K_4'(n_1n_2)\,\mathscr{P}_{23}\left(\frac{n_2c_3b}{r(n_2c_3)\,r(c_3b)\,r_0}\right)dF(c_3)+\int_{V_2} K_4'(n_1n_2)\,\mathscr{P}_{44}\left(\frac{n_2n_3b}{r(n_2n_3)\,r(n_3b)\,r_0}\right)dV(n_3), \tag{16}$$

where the known functions are determined according to the following scheme

$$K_{11}'\left(c_1c_2M/r(c_1c_2)\,r(c_2M)\,r_0\right)=K_1'(c_1c_2)\,K_1'(c_2M), \tag{17}$$

$$K_1'(c_1c_2)=K_1(c_1c_2)\,R\left(\frac{c_1}{r(c_1c_2)}\right)\gamma\left(\frac{c_2}{r(c_1c_2)\,r(c_2M)}\right)\exp\left(\int_{r(c_1)}^{r(c_2)}-k(r)\,dr\right). \tag{18}$$

An advantage of systems (9)-(12) and (13)-(16) is that each is a closed one and has a unique solution in terms of the sought opticogeometrical functions $\mathscr{P}_{ik}$ (i, k = 1, 2, 3, 4). These systems of integral equations can be solved on a computer [9].

On the other hand, the solution to Eqs. (1), (2) for the luminance of effective radiation $B_e(m/r_0)$ or $B_e(b/r_0)$ along a given direction r_0 and coming respectively from an arbitrary boundary surface element $dF(m/r_0)$ or from an arbitrary medium volume element $dV(b/r_0)$ can be put in the form [9]

$$B_e\left(\frac{M}{r_0}\right)=B_c\left(\frac{M}{r_0}\right)+\int_F B_c\left(\frac{c_1}{r_1}\right)\mathscr{P}_1(c_1M)\,dF(c_1)$$

$$+\int_V B_c\left(\frac{n_1}{r_1}\right)\mathscr{P}_2(n_1M)\,dV(n_1), \tag{19}$$

$$B_e\left(\frac{b}{r_0}\right)=B_c\left(\frac{b}{r_0}\right)+\int_F B_c\left(\frac{c_1}{r_1}\right)\mathscr{P}_3(c_1b)\,dF(c_1)$$

$$+\int_V B_c\left(\frac{n_1}{r_1}\right)\mathscr{P}_4(n_1b)\,dV(n_1). \tag{20}$$

The opticogeometrical resolvents $\mathscr{P}_1(c_1m)$, $\mathscr{P}_2(n_1m)$, $\mathscr{P}_3(c_1b)$, $P_4(n_1b)$ representing the effects of innumerable dispersions respectively at the boundary surfaces and inside the medium volume are determined from the following system of opticogeometrical integral equations:

$$\mathscr{P}_1(c_iM)=L_1(c_1M)+\int_F \mathscr{P}_1(c_1c_2)\,L_1(c_2M)\,dF(c_2)$$

$$+\int_V \mathscr{P}_3(c_1n_2)\,L_2(n_2M)\,dV(n_2), \tag{21}$$

$$\mathscr{P}_3(c_1b)=L_3(c_1b)+\int_F \mathscr{P}_1(c_1c_2)\,L_3(c_2b)\,dF(c_2)$$

$$+\int_V \mathscr{P}_3(c_1n_2)\,L_4(n_2b)\,dV(n_2), \tag{22}$$

$$\mathscr{P}_2(n_1M)=L_2(n_1b)+\int_F \mathscr{P}_2(n_1c_2)\,L_1(c_2M)\,dF(c_2)$$

$$+\int_V \mathscr{P}_4(n_1n_2)\,L_2(n_2M)\,dV(n_2), \tag{23}$$

$$\mathscr{P}_4(n_1b)=L_4(n_1b)+\int_F \mathscr{P}_2(n_1c_2)\,L_3(c_2b)\,dF(c_2)$$

$$+\int_V \mathscr{P}_4(n_1n_2)\,L_4(n_2b)\,dV(n_2) \tag{24}$$

or

$$\mathscr{P}_1(c_1M)=L_1(c_1M)+\int_F L_1(c_1c_2)\,\mathscr{P}_1(c_2M)\,dF(c_2)$$

$$+\int_V L_3(c_1n_2)\,\mathscr{P}_2(n_2M)\,dV(n_2), \tag{25}$$

$$\mathscr{P}_2(n_1M)=L_2(n_1M)+\int_F L_2(n_1c_2)\,\mathscr{P}_1(c_2M)\,dF(c_2)$$

$$+\int_V L_4(n_1n_2)\,\mathscr{P}_2(n_2M)\,dV(n_2), \tag{26}$$

$$\mathscr{P}_3(c_1b) = L_3(c_1b) + \int_F L_1(c_1c_2)\,\mathscr{P}_3(c_2b)\,dF(c_2)$$

$$+ \int_V L_3(c_1n_2)\,\mathscr{P}_4(n_2b)\,dV(n_2), \quad (27)$$

$$\mathscr{P}_4(n_1b) = L_4(n_1b) + \int L_2(n_1c_2)\,\mathscr{P}(c_2b)\,dF(c_2)$$

$$+ \int_V L_4(n_1n_2)\,\mathscr{P}_4(n_2b)\,dV(n_2). \quad (28)$$

Here the known functions are determined from the following formulas

$$L_1(c_1M) = K_1(c_1M)\,R\left(\frac{M}{r(c_1M)}\right)\gamma\left(\frac{M}{r(c_1M)\,r_0}\right)\exp\left(-\int_{r(c_1)}^{r(M)} k(r)dr\right), \quad (29)$$

$$L_2(n_1M) = K_2(n_1M)\,R\left(\frac{M}{r(n_1M)}\right)\gamma\left(\frac{M}{r(n_1M)\,r_0}\right)\exp\left(-\int_{r(n_1)}^{r(M)} k(r)\,dr\right), \quad (30)$$

$$L_3(c_1b) = K_3(c_1b)\,\beta\left(\frac{b}{r(c_1b)}\right)\gamma\left(\frac{b}{r(c_1b)\,r_0}\right)\exp\left(-\int_{r(c_1)}^{r(b)} k(r)\,dr\right), \quad (31)$$

$$L_4(n_1b) = K_4(n_1b)\,\beta\left(\frac{b}{r(n_1b)}\right)\gamma\left(\frac{b}{r(n_1b)\,r_0}\right)\exp\left(-\int_{r(c_1)}^{r(b)} k(r)\,dr\right). \quad (32)$$

We have here for the first time derived integral equations for the luminance of effective radiation for the general case of optically anisotropic boundary surfaces and media in a closed system, we have also obtained their solution in the form of opticogeometrical integral equations.

NOTATION

$B_c(c_1/r(c_1c_2)) = B_0(c_1)\varepsilon(c_1)\gamma_c(c_1/r(c_1c_2))$	is the luminance of intrinsic radiation from point c_1 in a system along direction $r(c_1c_2)$ (from point c_1 to point c_2);
$B_0(c_1)$	is the luminance of intrinsic radiation of a perfectly black body at the temperature of the given point c_1;
$\gamma_c(c_1/r(c_1c_2))$	is the indicatrix of intrinsic radiation from point c_1 along direction $r(c_1c_2)$;
$B_e(c_1/r(c_1c_2))$	is the luminance of effective radiation from point c_1 along direction $r(c_1c_2)$;
$R(c_1/r(mc_1))$	is the reflection factor at point c_1 from direction $r(mc_1)$;
$\gamma(c_2/r(c_1c_2)r(c_2c_3))$	is the indicatrix of dispersion at point c_1 from direction $r(c_1c_2)$ along direction $r(c_2c_3)$;
$r(c_i)$	is the space coordinate of point c_i;
$c_i \in F$;	
$m \in F$;	
$n_i \in V$;	
$b \in V$;	
V	is the volume of the medium;
F	is the boundary surface of the system;
$k(r(n_i))$	is the extinction factor of the medium at point n_i with coordinate $r(n_i)$;
$dF(c_1)$	is the element of boundary surface;
$dV(n_1)$	is the element of medium volume;
$\beta(n_2/r(c_1n_2))$	is the dispersion factor at point n_2 from direction $r(c_1n_2)$;
$\varepsilon(c_1)$	is the emissivity of the system c_1.

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